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Reflecting Prisms.

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Read 14th February, 1918, and discussed 14th March, 1918.

THERE are many purposes for which prisms may be used as reflectors in Optical Instruments, and the author has thought that it may be of interest to the Society to have a few facts and suggestions put before its members.

In many cases it is preferable to use a prism in place of mirrors. There is no need to silver the surface in some cases; they are often easier to mount, and for certain types of prism give a definite angle of deflection. The prismatic binocular is a case in point. If it had been necessary to mount four mirrors inside the tubes and keep them in proper adjustment, the prismatic binocular would not exist at the present day—except, perhaps, in a museum.

In this paper it is proposed to deal with prisms in which the reflections are all in one plane. The geometry of the ray-paths in the more general case, where the reflecting surfaces are inclined at all sorts of angles like the facets of a crystal, is exceedingly complicated to work, and still more difficult to illustrate by diagrams.

The reflections being all in one plane, and the geometry consequently two-dimensional, the prism will act as a simple mirror, or will deflect all rays through the same angle, according as whether the number of reflecting surfaces is odd or even.

Single Reflection Prisms.—The only one which calls for much comment is the one in the form of a right-angled isosceles triangle—the binocular prism—in which the slant face is used as a reflector. This prism can be used to invert a bundle of rays without deflection, or to deflect through various angles just as a plain mirror would. It has this advantage over the latter, that, owing to the refraction produced at the entrant face of the prism (which is nullified by corresponding refraction on emergence), it is possible to obtain reflection of a beam which comes in from behind the reflecting face. (Fig. 1, dotted line.)

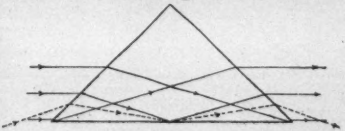

Fig. 1.

In order to get a pure reflection effect without any refraction, it is necessary that the two base angles of the prism should be equal, though not necessarily exactly 45°. If the image of the prism formed by reflection in the base be the dotted portion of Fig. 2, it is clear that the ray which is actually O A B C P would pass along O A B C′ P′ if we had a block of glass X N Y N′ with parallel faces X N and Y N′. If these two lines are not parallel —which is equivalent to saying that the base angles of the prism are not equal—the ray will emerge from C′ in a slightly different direction, C′Q′, and correspondingly the emergent ray through the prism would be C Q instead of C P.

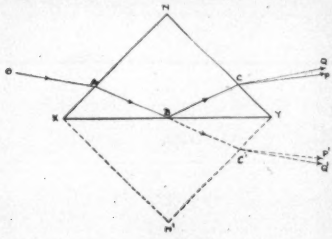

Fig. 2.

The deviation Q C P or Q′ C′ P′ can be expressed as

$$\phi\{\sec\theta\,(n^2-\sin^2\theta)^{\frac{1}{2}}-1\}$$

where θ is the angle which the ray O A makes with the normal to the face at A and ϕ is the difference of the base angles. This deviation increases rapidly with θ; for instance, it is twice as great at 60° as at 45°. Consequently it is not possible to use such a mirror for measurement of deflection in the way in which a sextant mirror is used unless the suitable correction is made at every point along the angle scale.

Possibly prisms might be manufactured so as to be reasonably isosceles, particularly as there is a very simple test for such a condition. If a distant point of light be viewed by very oblique reflection in the base of the prism, a second image can also be seen by reflection at the inside of the base. The base surface is, in fact, reflecting light from the object on both sides at once (Fig. 3). The two images will coincide only if the base angles are equal; alternatively they will be separated by an angle given by the formula above.

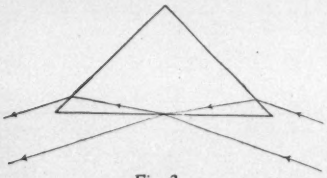
Fig. 3.

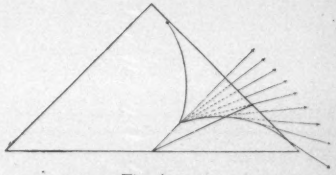
Fig. 4.

A prism of this type is used as an azimuth mirror with the Mariner's Compass, being mounted so as to rotate round a horizontal axis and to throw light from the sun down through a lens and so form an image of the sun upon the Compass card.

The position on which the axis is fixed relatively to the prism is of some importance in order that the maximum amount of light may be transmitted through the lens. The central ray of the bundle is clearly the one which is reflected at the middle point of the base, and the caustic of the emergent central ray is shown in Fig. 4. If it is desired only to use the prism with its base unsilvered, then the lower part of the caustic will be the portion with which we are concerned, and the centre of rotation should be an approximate centre of curvature of the lower portion of the caustic in question. On the other hand, if a wider range of use is required, and the surface is silvered, the centre of rotation would be better placed near the cusp of the caustic.

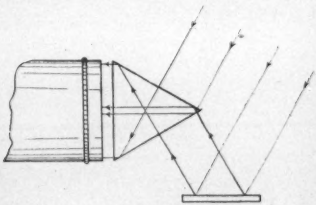

Fig. 5.

Another form of prism, in which the ray is reflected once only, is used by M. Driencourt in the Astrolabe bearing his name. This instrument is for the determination of the instant when a star reaches an altitude of 60° above the horizon, and the method by which this is done is shown in Fig. 5. Strictly speaking, this is not a single reflection prism in the sense that the previous one is. Rotation round an axis perpendicular to the plane of the paper will deflect the emergent beams by the same amount in each case, so that the two images of the star would remain in contact although they would move about in the field of view. Theoretically such a prism should be exactly equilateral, but from the point of view of the observer it might be a distinct advantage if the angles differed slightly from one another as in that case the prism could be used with each face successively vertical, and consequently three contacts between the two images of the star obtained instead of one. The mean of the time of the three observations would be exactly the instant at which the star's altitude was 60°.

Double Reflection Prisms.—All prisms in this class will deflect light through double the angle between the two faces, provided there is no refraction error due to bad design or bad workmanship.

The forms which such prisms may take vary considerably, and for the purposes of illustration the best forms for every 15° of angle from 0° to

90° are shown in Fig. 6. There are certain points connected with the design of all these which are identical, and may be referred to in a convenient form by the use of a strip of folded paper to represent the bundle of rays. If we take the first of the series it is clear that the path of the outside rays of the bundle is the same as the edges of a strip of paper the same width as the bundle folded at angles of 45° on the dotted lines AD and BC (Fig. 7). If the strip of paper is opened out it is clear that any ray

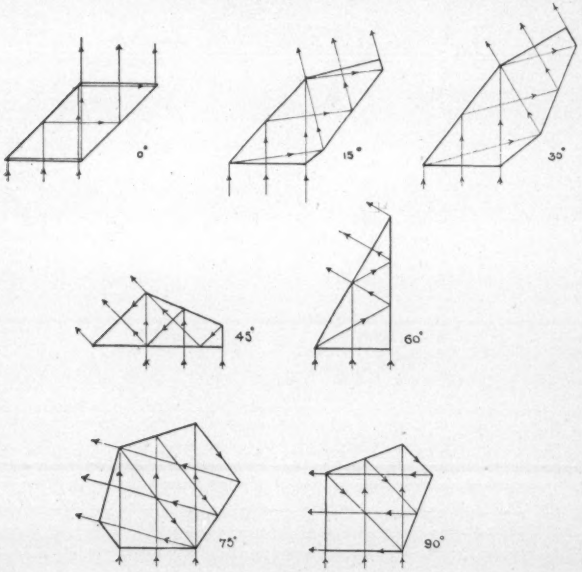

Fig. 6.

of light which crosses the entrant face between A and B and the emergent face between C and D will also pass through the prism. The limiting ray which can get through is that represented by the line A C, so that, for most purposes, it is desirable to design the prisms so that the ratio of the breadth of the strip to its length may be as large as possible.

It will be convenient, in fact, to denote the angle A C B as the aperture of the prism. The designs of prisms shown in Fig. 6 all aim at making

this aperture as large as possible. For the Zero deflection prism explained by Fig. 7 this angle is $\tan^{-1}\frac{1}{2}$ or approximately 26°.

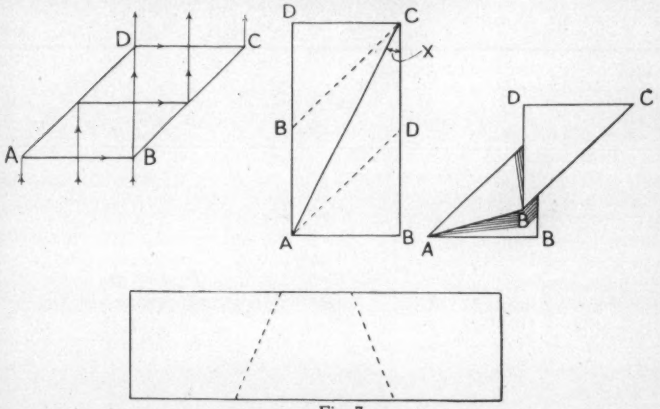

Fig. 7.

The corresponding folding for 90°, the optical square, is also shown in Fig. 7.

The first three prisms of the series are designed on the same plan of which the general type is shown in Fig. 8. The angle of deflection is 4α, and the suitable angles are all marked so as to give maximum aperture. For the 45° prism this form becomes unsuitable, as a more compact form can be obtained, one face acting both as transmitter and reflector of light. Curiously, the 60° prism follows the same form as shown in Fig. 8, but as, in such case, the first reflecting surface and the emergent face make a continuous surface, the aperture is really double that given by the general case above.

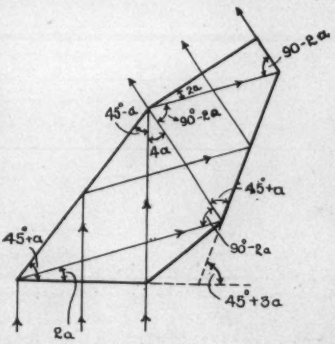

Fig. 8.

After 60° the method of arrangement has to be altered in order to avoid re-entrant angles, and the

apertures are reduced both with the 75° and the 90° prisms. It will be seen that in both cases there is symmetry with respect to a certain line, and consequently a maximum aperture.

Angles above 90° have not been all worked out in detail, but there are two types which possess maximum aperture for the same reason. These are 120° and 180°, which are illustrated in Fig. 9.

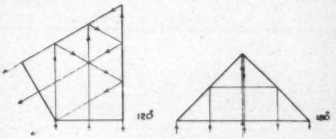

Fig. 9.

In all these prisms the entrant and emergent faces are placed at right angles to the beam so that the refraction errors which may arise owing to workmanship, will be a minimum. The aperture discussed above has in all cases been the aperture *in the prism.* This is not the same as the *external aperture*, inasmuch as the extreme ray which can pass through the prism will pass the entrant and emergent faces obliquely, and will consequently be refracted at these two places.

The values of the internal and external apertures are tabulated below, the index of the glass being taken as $n_D = 1{\cdot}5180$.

	Internal Angular Aperture.	External Angular Aperture.
0°	26½° ..	42½°
15°	23¾° ..	37½°
30°	21° ..	33°
45°	30½° ..	50¼°
60°	30° ..	49½°
75°	13¼° ..	20¼°
90°	16¼° ..	25¼°
120°	16° ..	25°
180°	26½° ..	42½°

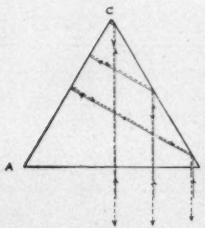

Fig. 10.

Triple Reflection Prisms.—If light traverses one face A B (Fig. 10) of an equilateral triangle A B C normally, it can be reflected at C B, then at A C, a third time at C B and will retrace its original path. To do this the side A C should be silvered, although a faint reflection could be obtained with an unsilvered surface. This affords a method of measuring the errors of a 60° prism. If the two base angles are $60 - \alpha$ and $60 - \beta$ and an auto-collimating telescope be placed in front of the face A B, three images will be obtained. Firstly the direct reflection from A B, secondly

the image by triple reflection as above, and thirdly by triple reflection starting from A C.

The separations of two and three from one will be $2n(\alpha+2\beta)$ and $2n(2\alpha+\beta)$. If these are measured, α and β can be obtained.

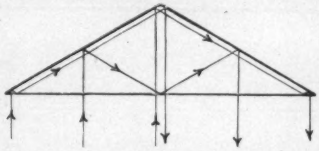

Fig. 11.

Another form of triple reflecting prism which might be of considerable use is one whose angles are 30° 30° and 120° (Fig. 11). The bundle of rays incident normally on the long side will emerge parallel and clear of the entrant bundle, a small deviation from the normal will, of course, involve an equal deviation, on the opposite side, of the emergent bundle. Such a prism might take the place of the horizon mirror in a sextant, which has to be set at an angle of about 75° with the telescope's axis, in order that the light between the two mirrors may not be cut off by the telescope tube.

By using such a prism in place of the mirrors the aperture could be very considerably enlarged, practically trebled. Further, the range of angles which can be measured by the sextant, now extending to 150°, could be increased.

The Detection of "Ghosts" in Prisms.

By T. SMITH, B.A.

(From the National Physical Laboratory.)

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THE term "Ghost" in connection with optical instruments means simply the presence of a quantity of light within the field of view which does not go to form the image which the instrument is intended to produce. This light may or may not form a definite image, and it may arise from objects within the proper field of view or it may come from a quite different direction. A second image of the proper view displaced slightly sideways in consequence of the incorrect working of prisms would be a ghost. On the other hand blurred images due to the presence of aberrations are not to be included. The method of detecting ghosts described below may perhaps be well known among optical manufacturers, but so far as the author knows no description of it has been published, and he is not aware that it has been used. Strictly speaking, it is not intended for the detection of ghosts, but for predicting whether the forms and the dimensions of prisms are such as to avoid the presence of ghosts. It is always easy to remove such unwelcome appearances by a considerable increase in the dimensions of the prism in which they arise, but quite apart from the difficulty of obtaining large pieces of glass of good quality, there are almost always serious objections to the employment of larger prisms than are absolutely necessary.

Ghosts nearly always arise through the occurrence of reflexions in ways that are not intended. When these reflexions take place in lenses no ghost is formed unless the ghost light has been reflected twice. Since the proportion of light reflected at each surface is small, such doubly reflected light is not often of sufficient intensity to become troublesome. With prisms a single additional reflexion may suffice to cause a ghost, and the effect is consequently much more serious. Moreover, when a prism is the first optical element of a system a single reflexion may superpose on the true image light coming from an altogether different angle; for example, an image of the Sun may be projected into the field of view and render vision almost impossible. The avoidance of ghosts in prism systems becomes consequently of the first importance.

The prisms which are most frequently employed in optical instruments are provided to deviate light without producing a spectrum or to erect an inverted image produced by a system of lenses. It is particularly with such prisms that the present note deals, though the method followed

is not necessarily confined to them. The prisms are therefore optically equivalent to a plane parallel block of glass. In designing optical instruments advantage is taken of this fact, in consequence of which the alteration of the directions of rays in space due to reflexion may be entirely disregarded. For many purposes a further simplification enables the block of glass to be entirely disregarded, it being replaced by an extension of the air separation equal to the thickness of the equivalent glass plate divided by its refractive index. Figs. 1, 2, and 3 illustrate

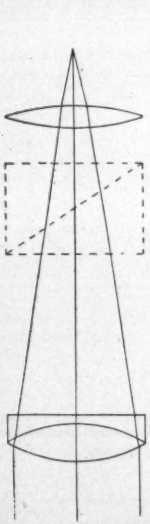


Fig. 1.

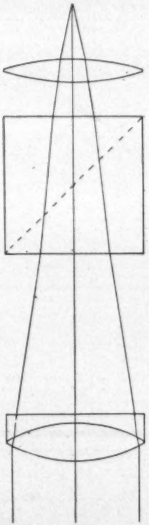


Fig. 2.

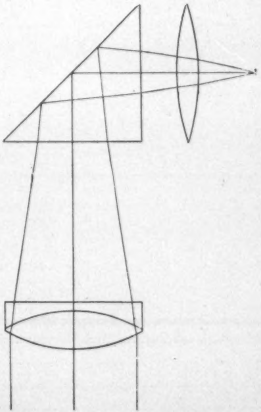


Fig. 3.

an original system, an equivalent "rectified" system, and a system with an equivalent air separation. In either of the two latter figures the complete path of any ray in the body of the prism is represented by a single straight line. This rectification so simplifies all questions in which prisms are involved that it should be used whenever possible. It will be shown that it may be applied very simply to show whether ghosts may arise in the use of prisms and if so under what conditions. For this purpose it is desirable that the method shown in the second illustration should be followed rather than the third, since in this case the actual

angles at which all rays meet the reflecting surfaces are retained. The diagrams will then show whether it is necessary to silver the reflecting surfaces, and as a rule the diagrams themselves are appreciably simplified.

In Fig. 4, ABC represents a principal section of an isosceles prism. PQ represents any ray inside the prism which is reflected at Q on BC to R on AC. Let BCD be the position of BCA determined by revolving the triangle about BC until it is again in its original plane. If PQ produced meets CD in S, the angles CQR and CQS are each equal to PQB, and thus the path QS is exactly equivalent to the path QR, and the complete path PS to the path PQR. Since the triangle is isosceles AB and CD are parallel and the prism is thus equivalent to a parallel block of glass.

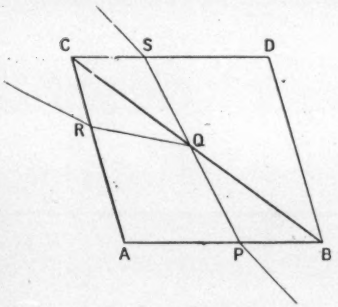


Fig. 4.

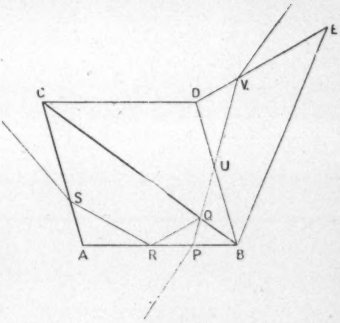


Fig. 5.

Fig. 5 shows another case which may arise in the same prism. The ray PQ after reflexion is deviated in such a direction that it meets the face AB at which it was incident before reaching AC. We may suppose that it is totally reflected at R on AB before being refracted out of the prism at S on AC. Fig. 5 shows the path PQRS rectified as PQUV. This may evidently be accomplished by rotating ABC about BC, the face at which the first reflexion occurs, to the position DBC, thus removing the change of direction at Q: and further rotating BCD about BD (which corresponds to AB the face at which the second reflexion takes place) to the position BED, by which means the second change of direction is eliminated. The complete path PQRS is thus replaced by its equivalent rectified path PQUV. The same principle may obviously be repeated indefinitely. If the triangle ABC is developed in a plane by successive rotation about its sides in all possible ways, straight lines will determine every possible path for all rays in the prism. A ray may be refracted

out whenever it meets a side developed in the way described within the critical angle.

It is convenient to consider rays travelling in directions opposed to those in which light will traverse the prism in use. In this way all possible directions of incidence which result in a given direction of emergence are found. Some of these directions of incidence may be left out of consideration, since the face on which the incidence takes place is the same as that of emergence, and as a rule no incident light can fall upon that face. In particular cases other incident rays may be neglected, because no incident light can reach the face owing to its being silvered or otherwise shielded.

In order that no spectrum should be produced, it is necessary for the lines in the diagram representing the equivalent positions of the faces of incidence and emergence to be parallel to one another. This condition by itself is not sufficient to ensure that all rays entering at a given face and emergent at another given face will have arisen from the same object

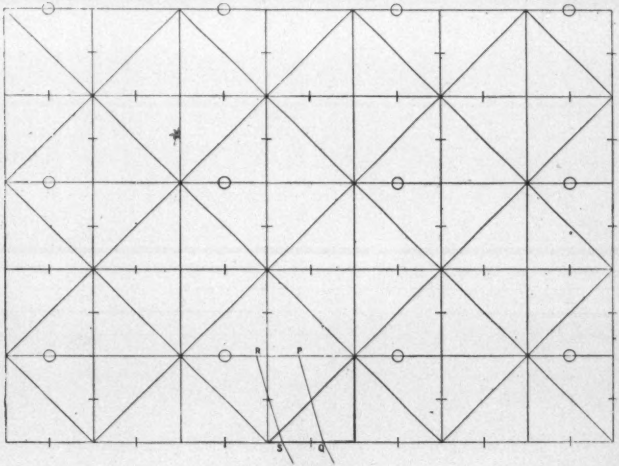


Fig. 6.

point when their final paths are coincident. The ordinary isosceles right angled prism illustrates this well. Fig. 6 illustrates its development in all possible ways. The original triangular section is denoted by thick lines, and the base and its various developed positions are marked by a

short transverse line. This may be regarded as the emergent face. All other lines parallel to this not so marked represent possible incidence faces. These may be divided into two classes, those in which an even number of reflexions has taken place between incidence and emergence, and those in which this number is odd. The number is determined by the number of lines crossed by the rectified ray between the two external faces considered. The incident faces corresponding to an even number of reflexions have been marked on this diagram with a small circle. All rays that have followed paths within the prism terminated at a face so marked are deviated by a constant amount, so that a rotation of the prism will not alter their final directions if their incident directions are unchanged. On the other hand, rays which have been reflected an odd number of times will have their deviation altered by twice the angle through which the prism has turned. It is thus clear that the same emergent direction in these two cases does not correspond to identity in the directions of the points of origin of the rays, and the inclusion of rays of both kinds involves considerable risk of ghosts. The two paths PQ and RS of this figure which are parallel and so correspond to parallel

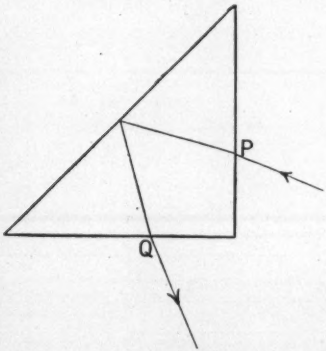


Fig. 7.

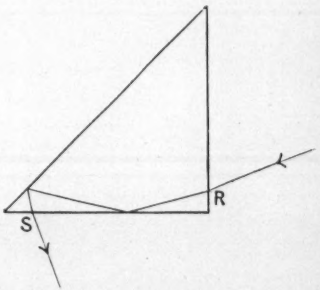


Fig. 8.

emergent rays, are illustrated in Figs. 7 and 8 as they actually occur in the prism.

Fig. 6 shows that the paths which have to be considered are equivalent to paths through glass blocks of thicknesses proportional to 1, 3, 5, 7, . . . The brightness of the image decreases rapidly with any advance along the series, and only the first will be of importance when the prism is not the first element of the optical train. When the prism comes first and is

in an exposed position the other reflexions may be of noticeable strength, but if the object is comparatively distant the various beams will be parallel if the prism is accurately made, and the multiple reflexions will be of no consequence. The diffraction effects due to the difference in the paths of the beams within the glass will not be noticeable in the strong general illumination of the field. In cases in which the beam within the prism is strongly convergent or divergent, the images corresponding to the longer paths within the prism will be sufficiently displaced from the plane of the primary image to cause no confusion to an observer. It is evident that the only rays which involve serious risks of ghosts follow a path such as RS, and also that to avoid the ghosts apertures must occur at some part of the system which will exclude a ray like RS while passing those typified by PQ. If the apertures and positions of the various lenses are all fixed this must evidently be accomplished by increasing the size of the prism until the incidence point of the extreme ray of any parallel pencil is included within a side of the diagram not marked with a circle.

The right angled isosceles prism may now be regarded as a constant deviation prism, the function it serves in a prismatic binocular. For the sake of clearness Fig. 9 has been drawn, though all the properties

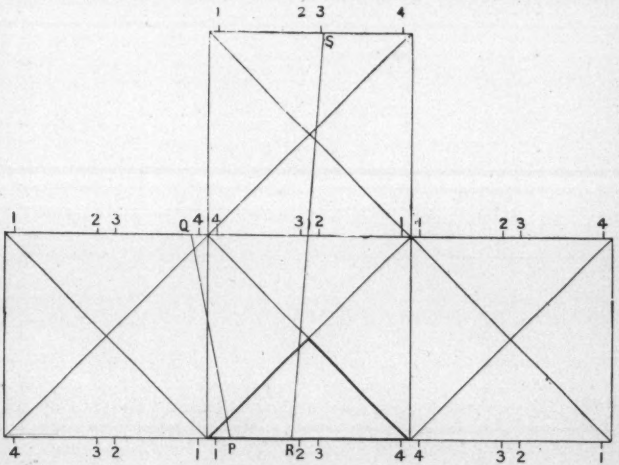


Fig. 9.

could be derived from Fig. 6. Four points, marked 1, 2, 3, 4, have been indicated on the hypotenuse to indicate the limits of the entrance and exit pupils of the prism. It is only necessary to consider rays which start from a point between 1 and 2 and end at a point between 3 and 4. The rays producing the normal image behave like rays incident normally on the hypotenuse and the glass thickness for such rays is equal to the length of the hypotenuse. The only rays which can cause a ghost and at the same time traverse a glass block of the same thickness are exemplified by PQ, which is reflected an odd number of times. If the inclination of this ray to the hypotenuse is not small, it will be refracted out of the system at one of equal faces of the prism and so will cause no trouble. Evidently the rays that are mischievous may be disposed of by the usual expedient of slitting the long face of the prism normally between 2 and 3, a small amount of generally scattered light then taking the place of a well defined ghost image. Another ghost may be caused by such a ray as RS, which has followed a path through the prism of twice the normal length. This may however be neglected since the image will be well out of focus.

Perhaps the prism which, after the right angled isosceles type, is most frequently used is the pentagonal prism which gives a constant deviation of a right angle. The development of this is shown in Fig. 10. As far as its optical properties are involved the prism is essentially quadrilateral; the truncation which results in the pentagonal form is indicated in the diagram by dotted lines. The long sides are silvered, so that reflexion necessarily occurs there. The emergent side AB and its images are distinguished by a short transverse line, and the incident face BC and its images by a small circle. The normal path is from FG to AB, with two reflexions, and therefore constant deviation. Obviously rays following paths to AB in directions which cut HJ and JK do not occur, since no incident light meets the face AB before BC. Rays in directions joining GH and AB evidently suffer total reflection at GH, and their further paths are obtained by repeating the whole diagram above H, or, what is equivalent, taking up the present diagram and moving it without rotation so that R takes up the position H. These rays thus ultimately terminate at faces corresponding to HJ or JK, and therefore do not exist. On the other side of the normal path a ray may strike EF and be there reflected; the continuation of the diagram may be represented by a translation of the diagram until P coincides with the original position of F. The ray will be refracted at the new position of PQ after three reflexions. A ghost therefore would be caused by such rays. Similarly a ray following a path from BC to BA causes a ghost; these are the only cases which can produce this result. The prism must therefore be so disposed that rays following paths from BC or EF are excluded from the final beams issuing from the instrument.

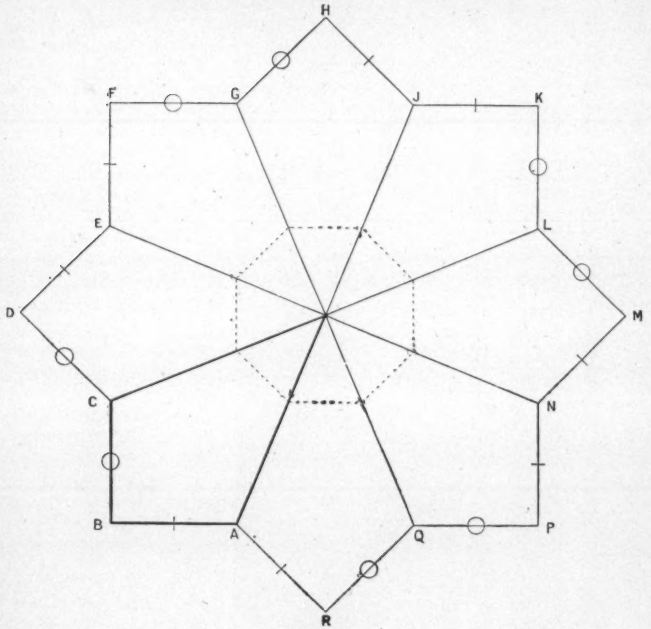


Fig. 10.

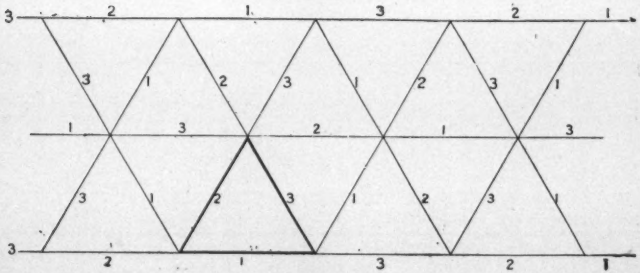


Fig. 11.

A few prisms of a less usual type may now be considered. Fig. 11 shows the development of the equilateral triangular prism. The faces have been marked by the figures 1, 2, 3. In some respects this is the ideal prism to use for the deviation of a beam of light. The diagram consists of horizontal and oblique lines, and the number of the latter meeting a horizontal at any point is always even. It follows that whenever the prism is equivalent to a parallel block of glass constant deviation can never occur. The practical limitations on its use result from the accident that our glasses have refractive indices less than 2. Fig. 12 shows the complementary prism, *i.e.*, the one which in combination with the preceding prism would make a right angled prism. Any prism and its complement have associated properties, though these may not at first be very evident. In Fig. 12 the face AC is assumed to be silvered. The normal ray is exemplified by OL, for which CD represents an image of CB. Although this diagram agrees with the former in the positions of the lines marked 1 and 3, there is an important distinction. In the

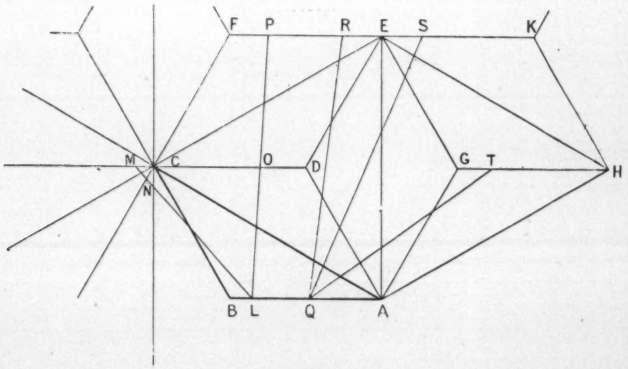


Fig. 12.

former case however the development occurs, a line in a position marked 1 is always an image of the original base line 1. On the other hand in the present diagram any given line such as EF will for some rays be an image of AB and for some rays an image of BC. If the development for the ray LOP is followed it is seen that the images of these two faces occupy in succession the following positions :—

Image of AB	Image of BC
AD	DC
ED	DC
EF	FC

On the other hand the development for such a ray as QR is

Image of AB	Image of BC
AD	DC
AD	DE
CD	DE
CF	FE

Thus for LOP, the face EF is an image of AB, while for QR the same face is an image of BC. Rays emergent normally at AB are thus represented by either OL or by QR. These have followed paths of quite different lengths; the latter group are deviated by a constant amount, the former group by an amount which varies with the position of the prism. The use of such a prism would thus cause complete confusion. For oblique rays such as QS and QT, it is easily seen that EK and GH represent images of BC. In the former case the number of reflexions is odd, and in the latter even. Since however the latter ray crosses DA at a small angle, the light would be refracted at the face AB with a spectrum, the system being equivalent to an ordinary equilateral prism as used in the spectroscope. Similarly the point M for the ray LM is on an image of BC, but the ray instead of suffering constant deviation would actually be refracted into the prism at N and a spectrum would be produced.

Figs. 13, 14, 15 illustrate two prisms of special interest, the isosceles prism with the angle opposite the base 45°, and its complement. Con-

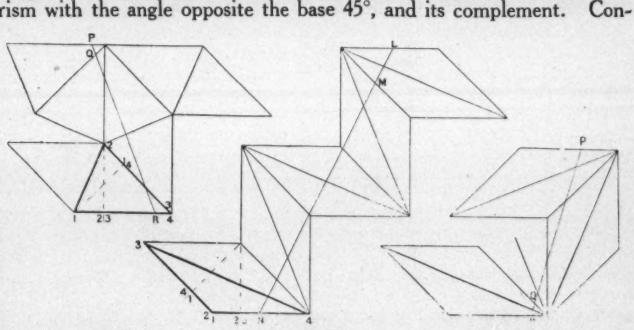


Fig. 13. Fig. 14. Fig. 15.

sidering first rays emerging from the base normally, it is seen that in both cases all these rays have been incident as a parallel beam on the

incident face, and that no constant deviation is possible. The base may however be divided into two parts as shown by the dotted line between 2 and 3. Corresponding to the emergent points 1, 2, 3, 4 arranged in order there are the incident points 1, 2, 3, 4 but arranged in the order 3, 4, 1, 2. Thus the extreme incident rays are brought together on emergence. Moreover the length of the path in the glass for the emergent portion 3 to 4 is $(1+\sqrt{2})$ times that for the portion 1 to 2. The prism could thus only be used in an ordinary optical instrument in positions where the light from a given point of the object examined was strictly parallel. It would however have other fields of application in physical optics, for instance in producing diffraction spectra of very high orders. It appears to possess the advantage over other pieces of apparatus designed for this purpose of being much less costly to construct, and should require much less glass than such an instrument as Michelson's Echelon. It would of course have to be constructed of very good glass, but the amount of work involved would not be great. It will be noted that so long as the prism is isosceles the angles may depart somewhat from the values shown. Perhaps the chief difficulty in the use of the prism in physical researches would be in the order of correction of the collimator employed, owing to the juxtaposition of rays from the opposite ends of the diameter of the objective. The simplest way of avoiding this would be to use two such prisms in succession; the path difference would then be doubled. A further disadvantage is the necessity for silvering or otherwise treating the reflecting face of the prism.

Another use for such a prism is the division of a given parallel beam into two widely separated but strictly parallel beams.

Fig. 13 shows a ray PQR which has not followed the normal path. If reflection took place at Q the deviation would be constant, thus causing a ghost. As drawn the ray would be refracted at Q, which is on an image of the emergent face, so that no light would enter the instrument in this direction. In the case of the complementary prism Q would be on the incident face, and a ghost would occur.

Fig. 14 shows the way in which the diagram may be repeated for a few special rays. L and N are on opposite faces, and the number of reflexions is odd. In most cases there will be refraction instead of reflexion at such a point as M. In the 135° prism M is on the incident face, and a ghost spectrum is formed: but in the 45° prism this point is on the emergent face, and no ghost occurs. Fig. 15 shows another ray which may give rise to a ghost. P is on the incident face, and the number of reflexions is even. If refraction occurs at Q, QR will result in a ghost spectrum in the 45° prism, but not in the other since in that case Q is on the emergent face. The three figures illustrate prisms in which the positions of the lines obtained in developing the diagram vary with the

direction of the ray. It is only in a few special cases such as the prisms developed in Figs. 6, 9, 10, 11, 12 that all methods of development result in the same outline diagram.

It may be useful to express algebraically some of the results to which the diagrams lead in the case of isosceles triangular prisms. The two equal faces may be assumed to be of length l, and the third side, which may be called the base, is supposed to be silvered. The two allied forms of prism may be taken together, the acute form having angles

$$\alpha, \quad \frac{\pi - \alpha}{2}, \quad \frac{\pi - \alpha}{2},$$

and the obtuse form angles

$$\pi - \alpha, \quad \frac{\alpha}{2}, \quad \frac{\alpha}{2}.$$

Let n be the integer determined by the conditions

$$2n\ \alpha \leqslant \pi \leqslant 2\ (n + 1)\ \alpha.$$

The properties of the prisms for rays incident normally on one of the equal faces may be simply described by supposing these faces divided up into $n+2$ parts. Number the sections from the base towards the vertex in the acute prism, and in the reverse direction in the obtuse prism. The length of section p is

$$l\{\cos\ (p-1)\ \alpha - \cos\ p\ \alpha\}$$

if $p \not> n$, and normal rays incident on this section behave as though they were passing through a parallel block of glass of thickness

$$l\{\sin\ (p - 1)\ \alpha + \sin\ p\ \alpha\}$$

unless α is so small that the rays are refracted out of the prism earlier. The number of reflexions which will have taken place is $2p - 1$ for the acute prism and $4p - 3$ for the obtuse, and is thus essentially odd in both cases. Rays incident on the last section behave in a similar way if $n+1$ is written for p in the above formulae except in that for the length of the section. The length of section $n+2$ is

$$l\{\cos\ n\ \alpha + \cos\ (n + 1)\ \alpha\}$$

provided this is positive; otherwise it is zero. Thus rays incident on any of the sections 1, 2, 3, ... n, $n+2$ will not result in achromatic ghosts of the constant deviation type. Rays incident on the $(n+1)^{\text{th}}$ section behave quite differently. The length of this section is

$$-\ l\ \cos\ (n + 1)\ \alpha$$

and the equivalent piece of glass when the reflexions are rectified is in general not parallel but prismatic. The angle of the equivalent prism and other particulars are given below.

Angle.	Number of Reflexions.		Face of Emergence.	
	Acute form.	Obtuse form.	Acute form.	Obtuse form.
$2n\alpha - \pi$	$2n$	$4n - 2$	Emergent	} Emergent
$(2n + 1)\alpha - \pi$	$2n + 1$	$4n$	Incident	
$2(n + 1)\alpha - \pi$	$2n + 2$	$4n + 2$	Emergent	

The first angle on this list at which the ray can be refracted out of the prism is to be selected. In the last two columns "Incident" and "Emergent" are used in the senses in which they apply to light passing through one of the regular $(n+1)$ sections. If α is an exact submultiple of a right angle this anomalous section does not exist for normal rays. In all other cases a ghost image is formed except for the acute prism having α an odd submultiple of two right angles, when the light entering the prism is refracted out at the same face and consequently does not traverse the rest of the optical system.

If the limiting points of the first n sections on the incident face are $a, a^1; b, b^1; c, c^1; \ldots n, n^1$; and the corresponding emergent points on a given ray are denoted by capital letters, the order of these points on the emergent face is $A^1, A; B^1, B; C^1, C; \ldots N^1, N$. If $p, p^1; q, q^1$ are the corresponding extremities of the $(n+1)^{th}$ $(n+2)^{th}$ sections, P^1 and Q coincide with the angular point of the prism remote from a and A^1. In the general case $P P^1$, unlike the other sections, is not equal to $p p^1$.

Rays within the prism which are inclined to the normal to the incident face may be treated in a similar way, first finding the value of an integer m for which $m\alpha$ and $(m+1)\alpha$ are limits within which the value of this inclination lies.

The illustrations which have been given of the method of finding every way in which a ray may be refracted through a prism are all of a simple character. The method may however be applied with ease to very complex and unsymmetrical prisms, and to cases in which several prisms have to be used in combination in order to avoid chromatic dispersion.

Owing to the brightness of many of the ghost images they can be photographed without difficulty. Fig. 16 shows a bookcase, desk and two pipes against a cream coloured wall which were selected as a subject for this purpose. Fig. 17 shows a photograph taken through an ordinary right angled prism which was placed in front of the lens, the camera having been turned approximately through a right angle. The main image (transposed left and right) is formed as in Fig. 7, and superposed

Fig. 16.

Fig. 17.

on this is a ghost of the bookcase and the attached diagrams formed as shown in Fig. 8. The difference in brightness of the two ends of the photograph is due to the limit of total reflexion being passed within the region included in the photograph. Fig. 18 shows a ghost image with the pentagonal prism illustrated in Fig. 10. The chairs and bookcase belong to the proper image, and the horizontally inverted desk and pipes are ghost images. Fig. 19 shows a ghost taken through a 60° constant deviation prism shaped like half an equiangular triangular prism. The glass of the prism was full of striae, which have affected the ghost much more than the erect image.

The last photograph, Fig. 20, is somewhat remarkable because it shows a ghost the presence of which might not have been anticipated, and which is hardly likely to have an opportunity of showing itself in a complete optical instrument. It is taken through a roof prism with the edge of the roof horizontal and on the left of an observer focussing the image on the ground glass. The camera is pointed at the fireplace which is about 120° to the left of the actual desk as seen from the camera. The bookcase, desk and pipes show up strongly in a ghost image formed by light which is incident on the front end of the prism at a very small angle (almost grazing incidence). The curved bright line which crosses the figure horizontally is another ghost—the image of a straight object. The fact that it is curved shows that it is not achromatic. The curvature of such an image is often the readiest indication that the equivalent block of glass so far as this image is concerned has not parallel faces. Very frequently the coloured edges of the image cannot be seen although curvature is decidedly pronounced.

In taking these photographs the prisms were carefully arranged in closely fitting dead black mounts which prevented any light entering the prism except through the one surface selected as the incident face. The mounts were extended to cover the lens also, so that all light entering the camera had previously passed through the prism. These precautions are essential if spurious ghosts are to be avoided.

DISCUSSION.

(Naval Instructor BAKER's and Mr. T. SMITH's Papers.)

Naval Instructor BAKER thought that Mr. Smith's method of developing a prism in all possible ways was excellent in the case of prisms of simple type, but that it would be difficult to apply to the case of more complex systems, such as rhomboid prisms, which would give rise to a maze of

Fig. 18.

Fig. 19.

triangulation. In practically all the cases shown the ghosts were due to the use of too small a prism.

Mr. S. D. CHALMERS said he had made use of the method of developing prisms to illustrate the path of beams of light through precious stones. By means of simple models it had been possible to trace the desirable changes to make in the cutting of the stones. In using Mr. Smith's method one could determine quickly which ghosts would be most undesirable, and thus limit the number of developments to be drawn to a few

Fig. 20.

cases. The more serious ghosts would be eliminated by the introduction of stops in positions indicated by the developed drawings.

Dr. R. S. CLAY said that in the case of three dimensional mirror problems, it was convenient to draw great circles on black or white spheres to represent the surfaces. The first time he had come across the method of developing prisms was in a paper by Dr. Drysdale, who had employed it for determining the sizes of prisms and the best angles to use in binoculars.

Mr. T. SMITH in reply to Naval Instructor Baker, said that increasing the size of the prism was one way of eliminating ghosts, and the drawings described would indicate the necessary size. Frequently this would lead to a much larger prism than would be necessary by adopting other methods, and so should not be taken as the only way of eliminating ghosts. In particular cases, as in a 30°, 30°, 120° prism enlargement of the prism would not be a remedy. Mr. Baker had not taken into account light entering the prism which was entirely unconnected with the object examined.

Mr. HUGHES said he had used a 30° prism on a sextant and no ghost was visible.

Mr. T. SMITH said a ghost would probably have been seen had a bright object, such as the sun, been suitably placed to provide a ghost within the field of view.

Mr. BAKER suggested that the best effective position for the stop was at the centre of the prism.

Mr. SMITH pointed out that this position would not exclude ghosts unless an unnecessarily large prism were employed. Generally speaking in the case of the simplest prisms, a stop should be provided at a real image of the first prism surface.

Gauge Projection Apparatus with Large Field of View Lens.

By E. M. EDEN, M.Sc.

(From the National Physical Laboratory.)

Read and discussed 14th March, 1918.

THE first projection machine used for gauge measurement at the National Physical Laboratory was for screw gauges and came into regular use for observing and measuring errors of thread form early in 1916. In this machine a compound system was used with microscope objective and eyepiece and a prism. The image of the screw thread was projected on top of an accurately drawn thread-form diagram drawn out exactly 50 times full size. The magnification was fixed at 50. With some difficulty a field of view of about $\frac{1}{2}''$ was obtained free from distortion.

The success of this method applied to screw threads caused a demand for a projection machine with a much larger field of view free from distortion to be used for the inspection and testing of plate gauges, many of which were of such forms that they were found very difficult to measure completely by ordinary mechanical methods.

The apparatus exhibited is the type which has been used for this work at the National Physical Laboratory for the last two years. A single projection lens of about $4\frac{1}{2}''$ focus is used with a field of view of about $2''$ diameter free from visible distortion with a magnification of 50 times. The plate gauge to be projected is illuminated with a parallel beam of light from a small arc lamp and a condenser of $6''$ or $7''$ focus.

The projection lens consists of a suitable photographic lens combined with a field lens, and it has been found possible by choosing suitable lenses to correct the distortion of the photographic lens by adjusting the distances between the two lenses. The particular example exhibited consists of a $6''$ Ross Xpres lens combined with a 3 part binocular objective of about $8''$ focus. The two lenses are mounted together with a separation found by experiment to give no perceptible distortion.

Owing to the necessary use of a parallel light beam, the field of view is limited by the diameter of the back component of the projection lens. The use of the field lens which is $2''$ diameter, increases the field of view to this size and largely improves the definition obtainable by using the photographic lens alone as well as correcting distortion.

Method of Observing Faults in Plane Silvered Mirrors.

By E. M. EDEN, M.Sc.

(From the National Physical Laboratory.)

Read and discussed 14th March, 1918.

In later screw thread projection machines the compound system first used was discarded in favour of a single projection lens of Petzval type. The projection is vertical, the projection lens being above the screw gauge, and the enlarged image is reflected on to a horizontal table from a horizontal silvered mirror near the ceiling. This mirror requires to be flat or distortion is introduced in the projected image. The bright background on which the shadow-like magnified image of the screw gauge is seen shows markings which are due to faults in the plane mirror.

Three types of markings have been noted—

(1) A more or less uniform mottling over the whole surface.

(2) Streaks and lines.

(3) A concentric system of grey circles with centre at the centre of the mirror.

All these faults have been observed on optically worked mirrors.

The system of concentric circles has only been seen in the case of one mirror, which is now being repolished and cannot therefore be shown.

Note :—The mirror showing the concentric grey circles has been repolished and it is reported that it still shows the same defect.

DISCUSSION.

Mr. T. Smith said that some of the defects of the mirrors, as shown up by the projection method, were not readily detectable on applying the ordinary tests for striae and other defects. Projection under the critical conditions realised in Mr. Eden's apparatus appeared to be an excellent way of showing up such defects.

Mr. S. D. Chalmers said the projection method was used some years ago at a rifle factory near Birmingham for examining the form of screw threads. The magnification was found to vary in comparing with the ultimate standards.

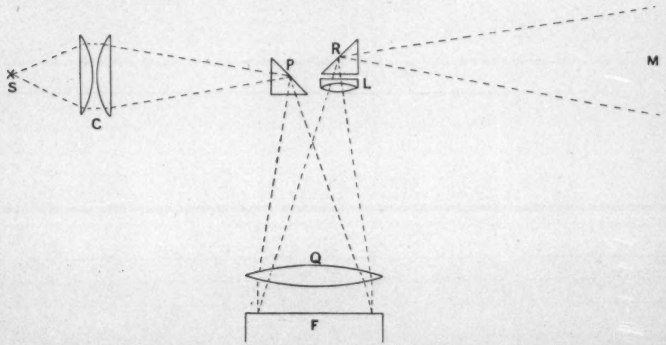
Mr. Eden in reply said that he was unaware of any regular use of the projection method for measuring gauges previous to 1916, when the method came into use at the N.P.L. As regards screw gauges at the N.P.L., the projection method was only relied on for thread form ; the screw gauge diameters being measured by micrometer screws.

Exhibit Showing a Change of Phase in the Light Reflected by a Surface on Silvering.

By Dr. REGINALD S. CLAY.

Read and discussed 14*th March*, 1918.

Dr. Clay said that at the Northern Polytechnic they had been making thin celluloid films and silvering them in order to obtain very light mirrors in connection with a research they were engaged upon. These they had made by the method first used by de la Rue of dropping a solution on water and allowing it to spread by capillarity over the surface of the water. They used a solution of celluloid in amyl acetate. At first they obtained reticulated films, but by the kindness of Mr. Glew—who had made these films for many years—they had got over that difficulty, which had been caused by the use of impure amyl acetate.



In silvering the films, which were so thin that they showed colour, it was noticed that the coating of silver changed the colour into its complementary. So that, for instance, a streak of colour which appeared red on an unsilvered film would become green when the film was silvered. To make the change obvious, a current of air was blown across the surface as the film was setting, which caused the film to show streams of colour. The films were picked up from the water by placing a ring cut from a large brass tube under the water, and lifting it slowly up in a

somewhat slanting direction. Then a part of the film was silvered, so that the same streak of colour was seen continued across both the silvered and unsilvered portions of the film.

Some eight or nine films were projected on to a screen by the method shown in the diagram. Light from an arc S of a lantern was focussed by the condenser C on a right angle prism P, by which it was reflected down a little obliquely on to a large double convex lens Q, 7″ diameter. The distance of P from Q was such that the light emerged from Q in a parallel beam. It then fell on and was reflected back by the coloured film F still as a parallel beam. Thus it was refocussed by the lens Q at about the same height above the lens as P. Here there was placed a simple achromatic lens, L, about 2″ diameter and of such a focal length (about 13″) that it formed an image on the screen, M, after reflection in the right angle prism R. The whole of the film was projected on the screen, and formed a blaze of colour several feet in diameter. The reversal of colour was clearly seen.

The films for the demonstration were prepared by Mr. E. H. Griffiths.

Exhibit Showing Effect of Cloth-Polishing a Glass Surface which had been Pitch-Polished.

By Dr. REGINALD S. CLAY.

Read and discussed 14th March, 1918.

THE surface of a prism which had been pitch polished was afterwards cloth polished, and the cloth polishing appeared to have removed the "Beilby layer" of glass which the pitch polishing had caused to flow over the lower unchanged glass, for it revealed all the pits and scratches which the pitch polishing had apparently removed, but had evidently only covered up with the molecules which had flowed over them. The surface was projected by the same apparatus. A second face which had been cloth polished over only part of its area was also projected, showing the undisturbed perfect polish of the pitch polished surface and alongside it the pits and scratches revealed on the part which had been subsequently cloth polished.

DISCUSSION.

Mr. S. D. CHALMERS said that French astronomers had succeeded in filming large mirrors. He thought that Fabry and Perot were responsible for the method employed.

Mr. F. HARRISON GLEW said that some films that he had made were exhibited by Lord Rayleigh about twenty years ago. Many years ago Messrs. de la Rue made beautiful non-solid films by dropping various varnishes on water and after drying sufficiently, raised them on cardboard. He did not think that they had succeeded in raising solid films on wires. The first films he himself had made were obtained by dropping glycogelatine on water. This gave very good films, but not quite so good as collodion. Black films can be made as large as 2″ square. Films over 1 ft. in diameter may be raised if a muslin net is placed beneath for support. By using as a support a zinc plate perforated with holes about $\frac{1}{8}$″ in. diameter, it is possible to make films twenty times thinner than a film showing the first blackness of Newton's order.

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